Shape Based Geometric Motion Planning for an Underactuated Highly-Articulated System

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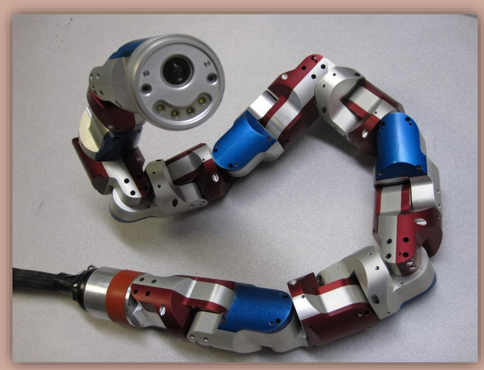
Introduction

Mechanical systems that use their internal shape changes to control their movements have always interested the geometric control community.

Snake robots are such systems that have many internal degrees of freedom and use these internal DOF's to control their movements.



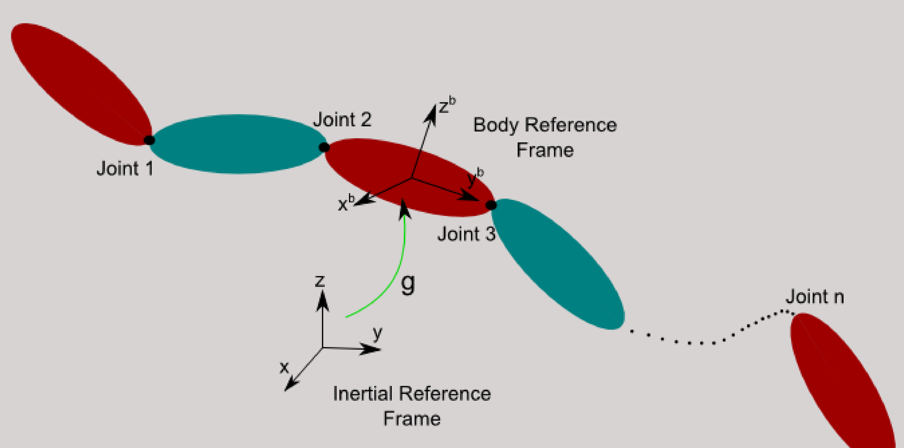
However, the high dimensionality of these systems makes it very difficult to control them.

We present a geometric solution to control highly articulated systems using "Shape Basis"

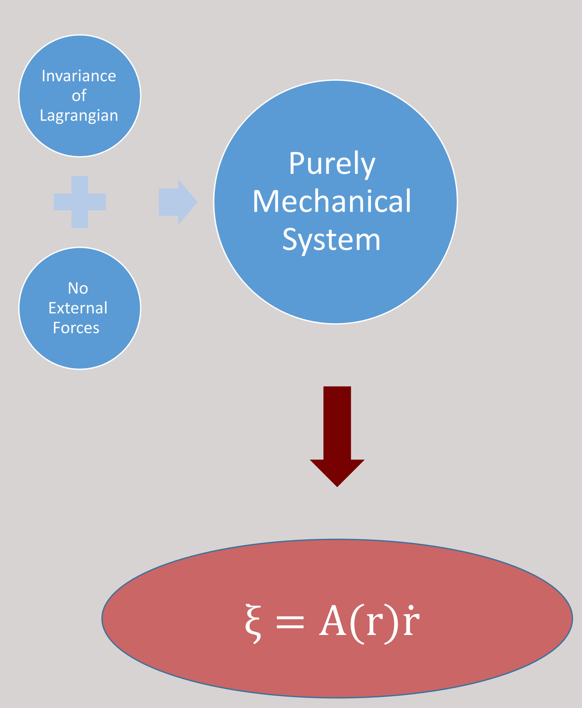
We show how we can benefit from the shape basis technique in order to generate gaits that move a mechanical system in a desired direction

We apply these techniques to a Snake Robot Floating in Space in order to generate gaits that reorients this snake in any direction.

Mathematical Model



- $g \in SE(3)$
- $\hat{\xi} = g^{-1}\dot{g}$ → Body Velocity
- $r \in \mathbb{R}^n$ → Base space (joint angles)



- A(r): Local Connection Matrix i.e. relates the body velocity of the snake to the joint velocities
- Interested in finding the change in orientation

→ Integrate ξ with time

$$\int \xi^i \, dt = \oint A^i(r) dr = \iint Curl\, A^i(r)\; ds$$

Shape Basis

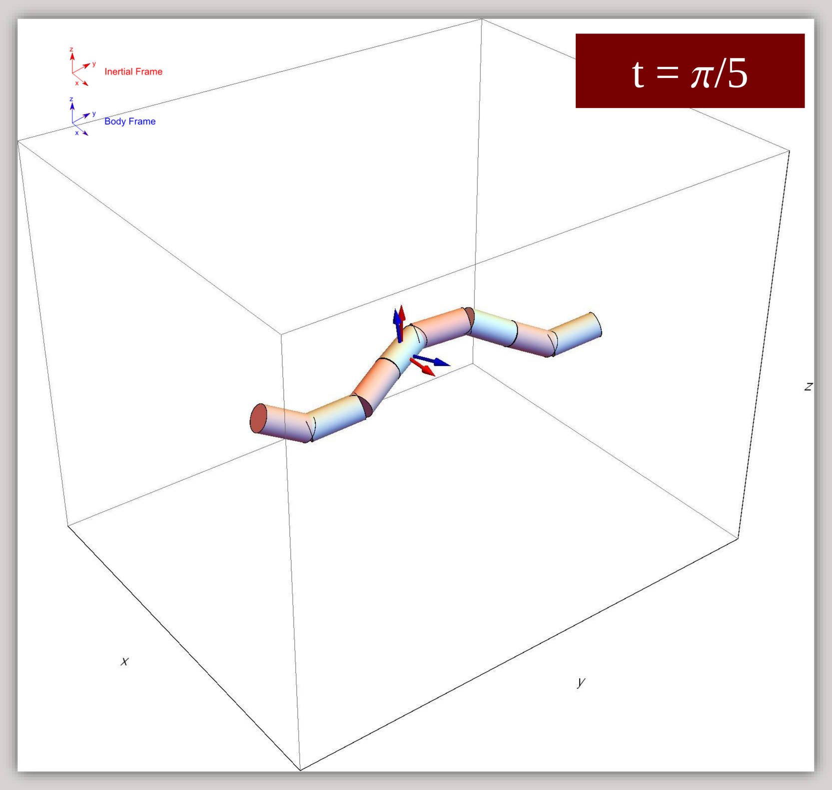
$r(n) = \theta(n) = \sin(wt + \Omega n) =$
$\cos(wt)\sin(\Omega n) + \sin(wt)\cos(\Omega n) =$
$\sigma_1(t)\sin(\Omega n) + \sigma_2(t)\cos(\Omega n)$

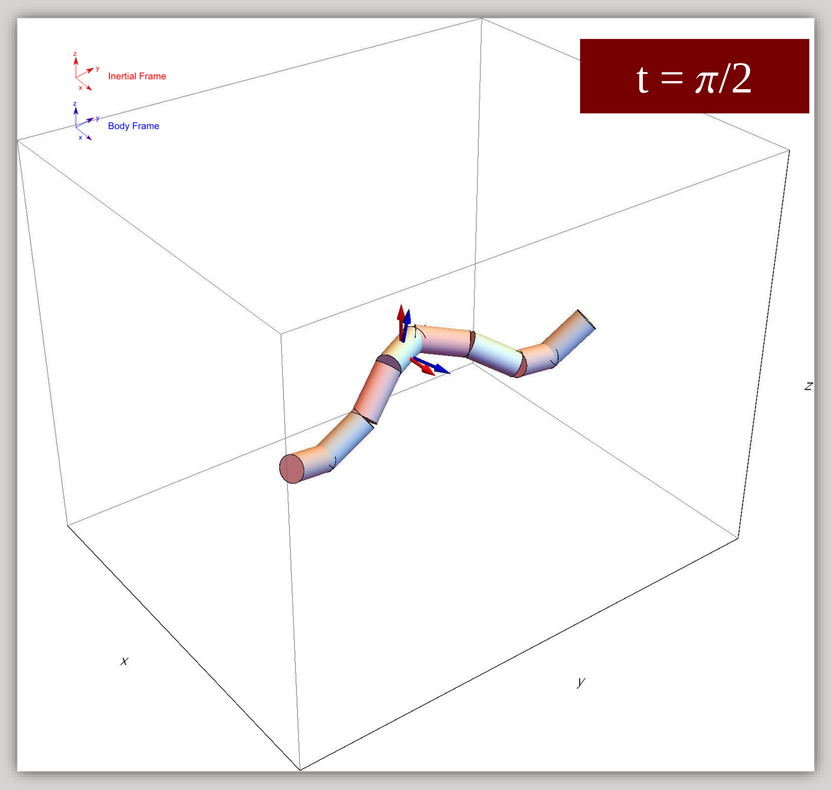
Shape Basis

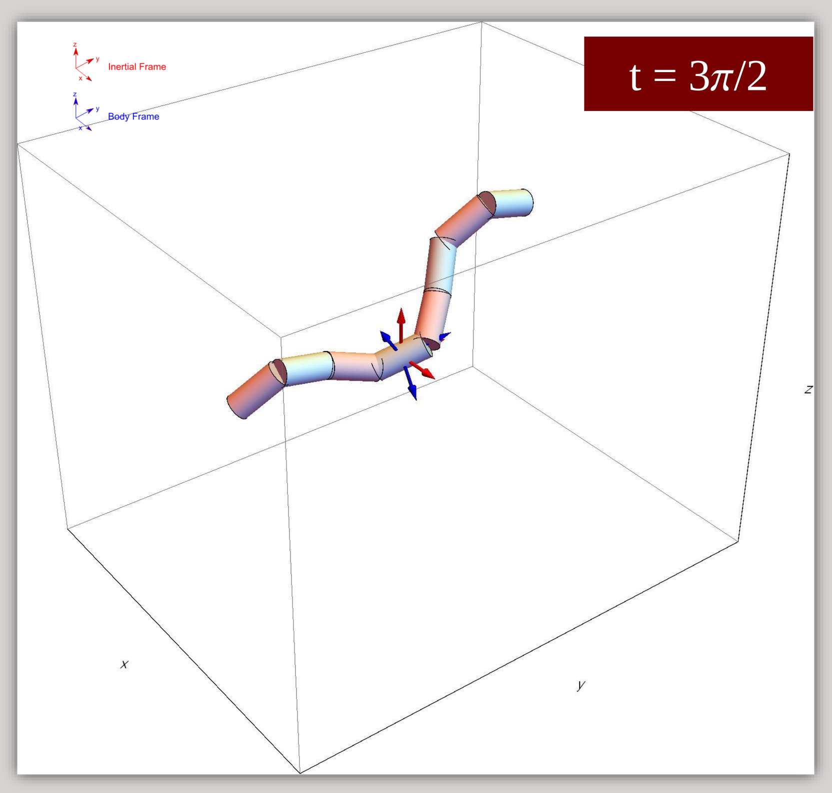
$$\sigma = \begin{pmatrix}\sigma_1 \\ \sigma_2\end{pmatrix} \in \mathbb{R}^2$$

$$\xi = \underbrace{A(r)\frac{\partial r}{\partial \sigma}}_{(3x2)}\dot{\sigma}$$

Example:
- $\theta(n)$ = angle of the n^{th} joint
- $\Omega = \pi/3$
- Shape Basis: $\{\sin(\Omega n), \cos(\Omega n)\}$
- $\sigma_1(t) = \pi/8\,(2\sin(t)+\sin(2t))$
- $\sigma_2(t) = \pi/8\,(2\sin(t)-\sin(2t))$



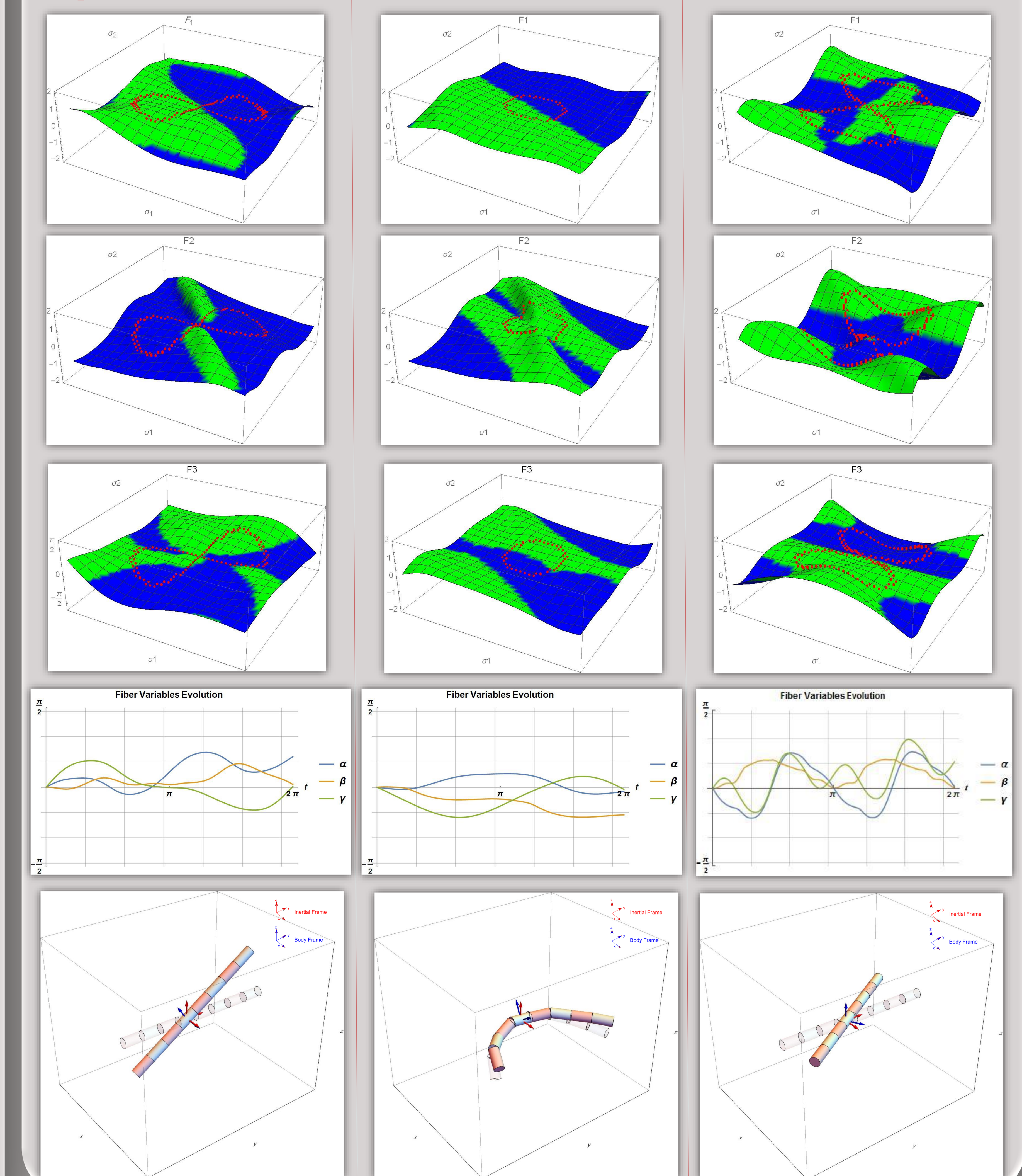




Simulation

- F_i = curl of the i^{th} row of the 3x2 matrix $A(r)\frac{\partial r}{\partial \sigma}$ → HEIGHT Function
- α: angle of rotation around inertial x axis
- β: angle of rotation around inertial y axis
- γ : angle of rotation around inertial z axis

Around X axis	**Around Y axis**	**Around Z axis**
$\Omega = \pi/3$	$\Omega = \pi/15$	$\Omega = \pi/2.6$
$\sigma_1(t) = \pi/8\,(2\sin(t)+\sin(2t))$	$\sigma_1(t) = 0.7\sin(2t)$	$\sigma_1(t) = 0.85\sin(4t)$
$\sigma_2(t) = \pi/8\,(2\sin(t)-\sin(2t))$	$\sigma_2(t) = -0.7\cos(2t)$	$\sigma_2(t) = \sin(2t)$



Conclusion

- Shape Bases reduces the dimensionality of the controlling Base space
- Using shape bases, we are able to easily generate gaits that reorient the Floating Snake Robot in any desired direction simply by looking at the three height function associated with our robot.

Future Work

- Optimizing the gaits for controlling the orientation of a Floating Snake using Minimum Perturbation Technique
- Optimization the choice of a shape basis
- Exploring more mechanical Systems that can be controlled using shape bases

References

1. Murray, R. M., Li, Z., & Sastry, S. S. (1994). *A Mathematical Introduction to Robot Manipulation.* CRC Press.
2. Travers, M., Hatton, R., & Choset, H. (2013). Minimum Perturbation Coordinates on SO(3). *American Control Conference.* Washington DC.
3. Shammas, E., Choset, H., & Rizzi, A. (2007). Geometric Motion Planning for Two Classes of Underactuated Mechanical Systems. *The International Journal of Robotics Research*, 1043-1073.